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Comment on “Inflation and flat directions in modular invariant superstring effective theories”*

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Abstract

The inflation model of Gaillard, Lyth and Murayama is revisited, with a systematic scan of the parameter space for dilaton stabilization during inflation.

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I. INTRODUCTION

The inflation model has proven to be a promising candidate for describing the early universe. It offers a very natural and elegant solution to the horizon and flatness problems in Big Bang cosmology. Unfortunately, its success generally relies on fine tuning some small parameters, and requires one or more scalar fields (inflavons) to roll slowly down a nearly flat potential.

In principle, a flat potential is not realistic in quantum field theory. Any flat potential at tree level will most likely be destroyed by radiative correction. However, with the aid of supersymmetry, such a flat direction may be protected by a nonrenormalization theorem. In [1] a model with the required flatness was constructed, based on the superstring-derived effective theory of [2], which utilizes nonperturbative string effects to stabilize the dilaton in the true vacuum. For inflation to be viable, the dilaton must also be stabilized during inflation. The analytic solution to the stabilization conditions used in [1] contains an algebraic error. In this article, we solve the equations numerically, which permits a systematic scan of the parameter space for viable solutions.

II. THE MODEL

The effective potential from orbifold compactification was presented in [1]. The Kähler potential K and the Green-Schwarz counter term V_{GS} were taken to be

$$K = G + \ln V + g(V), \quad G = \tilde{G} + \sum_A X_A, \quad V_{GS} = b\tilde{G} + \sum_A p_A X_A, \\ \tilde{G} = \sum_I \tilde{G}_I, \quad \tilde{G}_I = -\ln(T_I + \bar{T}_I - \sum_A |\Phi_{AI}|^2), \quad X_A = \exp\left(\sum_I q_I^A \tilde{G}_I\right) |\Phi_A|^2, \quad (1)$$

where $g(V)$ parameterizes nonperturbative string effects, V is a vector superfield whose scalar component $V_{\theta=\bar{\theta}=0} = \ell$ is the dilaton, and $b = 30/8\pi^2$ governs the beta function for E_8 . The T_I are the chiral multiplets containing the moduli. The Φ_{AI} are untwisted sector chiral multiplets, and the Φ_A are twisted sector chiral multiplets. The component Lagrangian was computed in [2]. Specifically, the scalar potential is given by

$$V = \frac{1}{16\ell^2}(\ell g' + 1) \left| u(1 + b_a \ell) - 4\ell W e^{K/2} \right|^2 - \frac{3}{16} \left| b_a u - 4W e^{K/2} \right|^2 \\ + \sum_A \left(\prod_I x_I^{q_I^A} \right) \frac{|Y_A|^2}{1 + p_A \ell} + \sum_I \frac{1}{1 + b\ell + \sum_B (1 + p_B \ell) q_i^B X_B} \times \\ \left[\left| A_I (2\xi(t_I) x_I + 1) - e^{K/2} \sum_A \phi_{AI} W_{AI} \right|^2 + x_I \sum_A \left| W_{AI} e^{K/2} + 2\xi(t_I) A_I \bar{\phi}_{AI} \right|^2 \right] \quad (2)$$

where b_a governs the β -function for the condensing gauge sector,

$$A_I = e^{K/2} \left(\sum_\alpha q_I^\alpha \phi_\alpha W_\alpha - W \right) - \frac{u}{4}(b - b_a), \quad (3)$$

and

$$Y_A = e^{K/2} [W_A + K_A W] + \frac{u}{4} (p_A - b_a) K_A. \quad (4)$$

A. Vacuum conditions

In the true vacuum, all matter fields vanish. Hence $W = W_a = 0$. Recall that $K_a = (\prod_I x_I^{-q_I^a}) \bar{\phi}_a$, which vanishes in the vacuum as well. This means

$$Y_A = 0, \quad A_I = -\frac{u}{4}(b - b_a), \quad x_I = t_I + \bar{t}_I = 2\text{Re}t_I, \quad (5)$$

and the scalar potential reduces to

$$V_0 = \frac{1}{16\ell^2}(\ell g' + 1) |u(1 + b_a \ell)|^2 - \frac{3}{16} |b_a u|^2 + \sum_I \frac{1}{1 + b\ell} \left| \frac{u}{4}(b - b_a)(2\xi(t_I)x_I + 1) \right|^2. \quad (6)$$

Minimizing with respect to t_I , we obtain $2\xi(t_I)x_I + 1 = 0$. Therefore, in the vacuum ¹

$$V_0 \propto \frac{1}{b_a^2 \ell^2}(\ell g' + 1)(1 + b_a \ell)^2 - 3 = \frac{1}{b_a^2 \ell^2}(f - f'\ell + 1)(1 + b_a \ell)^2 - 3. \quad (7)$$

Now we need to find f such that

1. The dilaton is stabilized ($\partial V_0 / \partial \ell = 0$, $\partial^2 V_0 / \partial \ell^2 > 0$), and
2. the cosmological constant vanishes ($V_0 = 0$).

From these two conditions, we arrive at the following constraints:

$$\begin{aligned} 2(f - f'\ell + 1) + \ell^2 f''(1 + b_a \ell) &= 0, & f''' \ell^2(1 + b_a \ell) + 3b_a f'' \ell^2 &< 0, \\ (f - f'\ell + 1)(1 + b_a \ell)^2 - 3b_a^2 \ell^2 &= 0, \end{aligned} \quad (8)$$

where $\ell = \langle \ell \rangle_0$ is the *vev* of the dilaton in the vacuum.

¹The nonperturbative string effects are parameterized by two functions f and g , which are related by

$$\ell g' = f - \ell f', \quad g(\ell = 0) = f(\ell = 0) = 0$$

B. Inflation

To construct a model of inflation, we make the following assumptions [1].

1. $V^{1/4} \gg \sqrt{u}$.
2. $W \sim 0$.
3. $W_\alpha = 0$, except for $\alpha = C3$, which is in the untwisted sector.
4. All matter field vev 's are negligible.

Then the scalar potential during inflation is

$$V_i = \frac{\ell e^g}{(1 + b\ell)x_1 x_2} |W_{C3}|^2. \quad (9)$$

It is expected that W_{C3} has a power law dependence on the dilaton, which will be discussed later. The dilaton dependence of V_i can be written as

$$V_i = \frac{\ell^d e^g}{(1 + b\ell)}. \quad (10)$$

Once again, we need to stabilize the dilaton. This time, there is an extra constraint. That is, the dilaton vev during inflation is located in the domain of attraction of the true vacuum. Dilaton stabilization equations are

$$f - f'\ell + d - \frac{b\ell}{1 + b\ell} = 0, \quad f'' + \frac{1}{b\ell(1 + b\ell)^2} < 0. \quad (11)$$

C. Summary of the equations for dilaton stabilization

The stabilization equations are most simply expressed in terms of the rescaled dilaton field $\zeta = b\ell$. In terms of this variable they take the following form.

$$1. \text{ Vacuum: } \zeta = b \langle \ell \rangle_0$$

$$f'' + \frac{6\gamma^2}{(1 + \gamma\zeta)^3} = 0, \quad f - f'\zeta + 1 - \frac{3\gamma^2\zeta^2}{(1 + \gamma\zeta)^2} = 0, \quad f''' - \frac{18\gamma^3}{(1 + \gamma\zeta)^4} < 0, \quad (12)$$

where

$$b = \frac{30}{8\pi^2}, \quad \gamma = b_a/b. \quad (13)$$

2. Inflation: $\zeta = b \langle \ell \rangle_i$

$$f - f'\zeta + d - \frac{\zeta}{\zeta + 1} = 0, \quad f'' + \frac{1}{\zeta(1 + \zeta)^2} < 0. \quad (14)$$

For simplicity, we will use only the two leading terms for the nonperturbative parameters [3].

$$f(\zeta) = B \left(1 + A \sqrt{\frac{a}{\zeta}} \right) e^{-\sqrt{a/\zeta}}, \quad (15)$$

where A , B and a are adjustable parameters. As opposed to the previous equations, all derivatives that appear in these equations are with respect to the rescaled dilaton ζ .

III. PHENOMENOLOGICAL CONSTRAINTS ON THE PARAMETERS

A. The parameter γ

The effective gauge coupling at the string scale is $g^{-2} = (f + 1)/2\ell$. Recall that the gravitino mass is given by

$$M_{\tilde{G}} = \frac{1}{4} b_a |\langle \bar{\lambda} \lambda \rangle|, \quad M_p = 1, \quad (16)$$

where M_p is the reduced Planck mass: $M_p = (8\pi G_N)^{-2}$. To establish the observed hierarchy, we want $M_{\tilde{G}} \sim 1 \text{ TeV}$. This determines the supersymmetry (SUSY) breaking scale:

$$M_{\tilde{G}} = \frac{1}{4} b_a \Lambda^3 / M_p^2 \sim 10^3 \text{ GeV}, \quad \Lambda \sim 10^{14} \text{ GeV}, \quad (17)$$

assuming $b_a \sim O(0.1)$. If SUSY is broken by a condensate, the renormalization group equation (RGE) tells us the scale Λ at which the gauge interaction becomes strong; in the leading log approximation

$$\mu \frac{\partial g}{\partial \mu} = -\frac{3}{2} b_a g^3, \quad \Lambda = M_p \exp(-1/3b_a g^2). \quad (18)$$

For $\mu = \Lambda \sim 10^{14} \text{ GeV}$, $3b_a g^2 \sim 0.1$. This relates γ to g^2 :

$$\gamma = \frac{b_a}{b} \sim .03 \frac{f + 1}{2\zeta}. \quad (19)$$

B. The parameter d

The D-term in the scalar potential contains a Fayet-Illiopoulos term:

$$V_D = \frac{g^2}{2} \left(\sum q_n K_n \phi_n + \xi_D \right)^2 \quad (20)$$

where $K_n \propto \bar{\phi}_n$, and $\xi_D \propto \ell$. This leads to a *vev* $\langle \phi_n \rangle \propto \ell^{1/2}$. This will in turn induce other *vev*'s of the form $\langle \phi_n \rangle \propto \ell^{-1}$ [1]. The superpotential in general has a power series expansion in all the matter fields. Since $V_i \propto |W_{C3}|^2$, we conclude that d is an *integer*, which may take on negative values.

IV. RESULTS

The equations (12)-(14) are solved self-consistently based on two input parameters: d and the gauge coupling g . The upper bound of d is determined by the inflation equation. In this case, there is no solution for $d \geq 2$. The lower bound of d is determined by the requirement that the dilaton remains in its domain of attraction. In the following table, the variables are defined as follows:

1. g_{max}^2 : the maximum value of g^2 such that the equations have solutions.
2. ℓ_0 : *vev* of the dilaton in vacuum.
3. ℓ_i : *vev* of the dilaton during inflation.

The RGE extrapolation of low energy couplings in the context of the Minimal Supersymmetric Standard Model (MSSM) gives $g^2 \sim .5$ at a scale of about $10^{16} GeV$. Unification at the string scale, $\mu_s = g$ in reduced Planck mass units, can be achieved [4] by adding additional matter fields. This increases g^2 , in some cases to a value as high as $g^2 \approx 1$. Hence we conclude that $d = 1$, $d = 0$ and $d = -1$ are candidates for a realistic model.

A typical solution is plotted here. Notice that in the scalar potential, an overall normalization proportional to the gaugino condensate is not included.

FIGURES

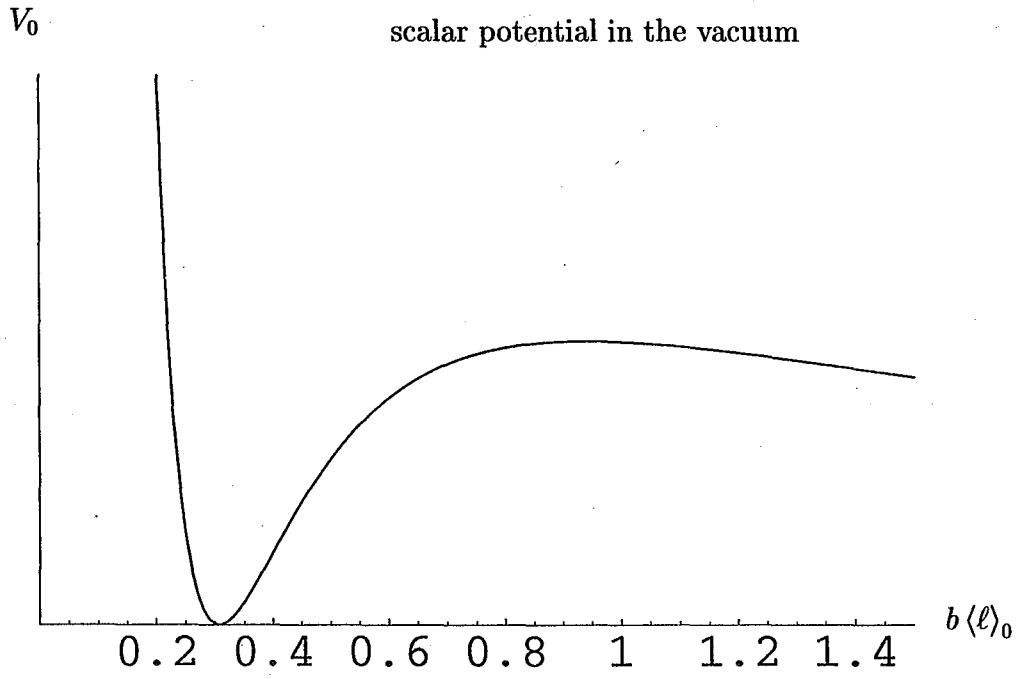


FIG. 1. Input parameters: $d = 1$, $g_{string}^2 = 1.46$

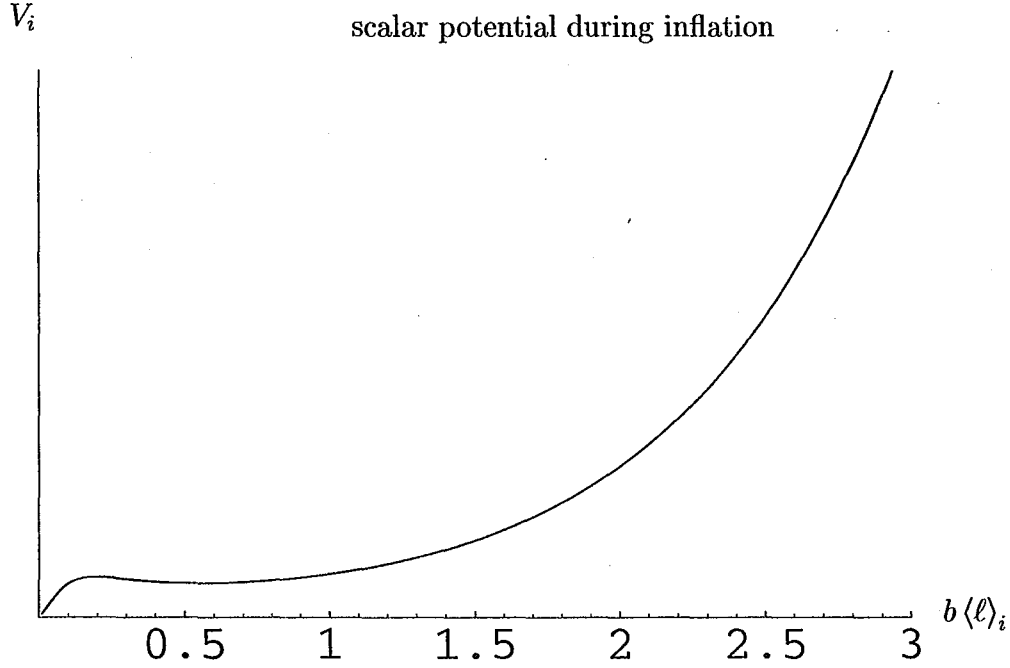


FIG. 2. Input parameters: $d = 1$, $g_{string}^2 = 1.46$

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TABLES

d	g_{max}^2	ℓ_0	ℓ_i	A	B	a
1	2.3	1.16	2.76	-.66	9.07	1
0	.73	.69	2.10	-.36	20.6	1
-1	.68	1.01	3.51	-.063	39.2	3
-2	.15	.22	.89	-.069	39.0	.65

TABLE I. Parameters for different values of d